MAT 2773, Probability and Statistics for Engineers

Assignment 3 - Solutions

[4] **3-174:** Let $N(t)$ be the number of inclusions in t millimeters cube. $N(t)$ has a Poisson distribution with mean $\lambda = 2,5\,t$.

(a) Here $\lambda = 2.5(1) = 2.5$. We want

$$P(N(1) \geq 1) = 1 - P(X = 0) = 1 - e^{-2.5}\frac{(2.5)^0}{0!} = 0.9179.$$

(b) Here $\lambda = 2.5(5) = 12.5$. We want

$$\begin{aligned} & P(N(5) \geq 5) \\ = \;& 1 - P(X < 5) \\ = \;& 1 - e^{-12.5}\left[\frac{(12.5)^0}{0!} + \frac{(12.5)^1}{1!} + \frac{(12.5)^2}{2!} + \frac{(12.5)^3}{3!} + \frac{(12.5)^4}{4!}\right] \\ = \;& 0.9947. \end{aligned}$$

(c) We want t such

$$0.99 = P(N(t) \geq 1) = 1 - e^{-2.5\,t}\frac{(2.5)^0}{0!}.$$

Alors,

$$t = \frac{\ln(1 - 0.99)}{-2.5} = 1.8420 \quad \text{mm}^3.$$

(d) We want to determine λ, the mean number of inclusions per mm^3, such that

$$0.95 = P(N(1) \geq 1) = 1 - e^{-\lambda}\frac{\lambda^0}{0!}.$$

We get

$$\lambda = \frac{\ln(1 - 0.95)}{-1} = 2.9957 \quad \text{inclusions}.$$

[4] **3-208:**

(a) Let X be the number of flaws in 50 panels. X has a Poisson distribution with mean $\lambda = (0.02)(50) = 1$. We want $P(X = 0) = e^{-1}1^0/0! = 0.3679$.

(b) Let X be the number of panels that need to be inspected before observing a flaw. X has a exponential distribution with parameter $\lambda = 0.02$. We want

$$E[X] = \frac{1}{\lambda} = \frac{1}{0.02} = 50 \quad \text{panels}.$$

(c) Let X be the number of flaws in one panel. X has a Poisson distribution with mean $\lambda = (0.02)(1) = 0.02$. The probability that a panel has at least one flaw is

$$P(X \geq 1) = 1 - e^{-0.02}\frac{(0.02)^0}{0!} = 0.0198.$$

Let Y be the number of panels, among $n = 50$, that have at least one flaw. Y has a binomial distribution with $n = 50$ and $p = 0.0198$. We want

$$P(Y \leq 2) = \sum_{y=0}^{2} \binom{50}{y} (0.0198)^y (0.9802)^{50-y} = 0.9234.$$

[4] **3-185:**

(a) The probability that no cars pass through the intersection in 30 seconds is

$$e^{-6(1/2)}\frac{[6(1/2)]^0}{0!} = 0.0498.$$

(b) The probability that at least three cars pass through the intersection in 30 seconds is

$$1 - e^{-6(1/2)}\left[\frac{[6(1/2)]^0}{0!} + \frac{[6(1/2)]^1}{1!} + \frac{[6(1/2)]^2}{2!}\right] = 0.5768.$$

(c) We want the smallest x such that $F_X(x) \geq 0.9$, where X has a mean of $\lambda = 6(1/2) = 3$. Since

$$F_X(4) = 0.8153 \quad \text{and} \quad F_X(5) = 0.9161.$$

Then, the minimum number of cars is 5.

(d) For a Poisson random variable, its mean and variance are equal. If the mean is 6, but the variance is 20, it is not reasonable to use a Poisson distribution to model the flow of traffic at this intersection.

[5] **4-4:**

(a)

$$P(X < 2) = \int_1^2 2\,x^{-3}\,dx = -x^{-2}\Big|_1^2 = 0.75$$

(b)

$$P(X > 5) = \int_5^\infty x^{-3}\,dx = -x^{-2}\Big|_5^\infty = 0.04.$$

(c)
$$P(4 < X < 8) = \int_4^8 x^{-3}\,dx = -x^{-2}\Big|_4^8 = 0.046875.$$

(d) $P(X < 4 \text{ or } X > 8) = 1 - P(4 < X < 8) = 0.953125$

(e)
$$0.95 = P(X < x) = \int_1^x u^{-3}\,du = -x^{-2}\Big|_1^x = \frac{-1}{x^2} + 1.$$

Then,
$$x = \left(\frac{1}{1 - 0.95}\right)^{1/2} = 4.4721.$$

[4] **4-18** Note that X is a continuous random variable, since F is a continuous function.

(a) $P(X < 1.8) = F(1.8) = 0.25\,(1.8) + 0.5 = 0.95$

(b) $P(X > -1.5) = 1 - F(-1.5) = 1 - [0.25\,(-1.5) + 0.5] = 0.875$

(c) $P(X < -2) = F(-2) = 0$

(d)
$$\begin{aligned} P(-1 < X < 1) &= F(1) - F(-1) \\ &= [0.25\,(1) + 0.5] - [0.25\,(-1) + 0.5] = -0.5 \end{aligned}$$

[4] **4-116** Let X the waiting time between consecutive log-ons (in minutes). X has a an exponential distribution with parameter $\lambda = 3$.

(a) We want $E[X] = 1/\lambda = 1/3 = 0.3333$ minute.

(b) We want $\sigma_X = \sqrt{V[X]} = \sqrt{1/\lambda^2} = \sqrt{1/3^2} = 0.3333$ minute.

(c) We want
$$0.95 = P(X \le x) = 1 - e^{-3\,x}.$$

Then,
$$x = \frac{\ln(1 - 0.95)}{-3} = 0.9986 \text{ minute.}$$

[4] **4-142**

(a) Let T be the time (in minutes) to form a packet, that is the waiting time for 5 messages. T has an Erlang distribution with $r = 5$ and $\lambda = 30$. Then,
$$E[T] = \frac{r}{\lambda} = \frac{5}{30} = 0.16667 \text{ minute.}$$

(b) We want

$$\sigma_T = \sqrt{V[T]} = \sqrt{\frac{r}{\lambda^2}} = \sqrt{\frac{5}{30^2}} = 0.07454 \quad \text{minute.}$$

(c) The event that a packet is formed in less than 10 seconds is equivalent to the event that there are at least 5 messages in 10 seconds. Let X be the number of messages in 10 seconds. X has a Poisson distribution with mean $\lambda = 30(10/60) = 5$. We want

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - e^{-5}\left[\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right] \\ &= 0.5595 \end{aligned}$$

(d) The event that a packet is formed in less than 10 seconds is equivalent to the event that there are at least 5 messages in 5 seconds. Let X be the number of messages in 5 seconds. X has a Poisson distribution with mean $\lambda = 30(5/60) = 2.5$. We want

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - e^{-2.5}\left[\frac{2.5^0}{0!} + \frac{2.5^1}{1!} + \frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!}\right] \\ &= 0.1088 \end{aligned}$$

[4] **4-212:** Let X be the diameter in inches. X has a normal distribution with mean $\mu = 0.002$ inch.

(a) We want σ such that

$$\begin{aligned} 0.9973 &= P\,(0.0014 < X < 0.0026) \\ &= P\left(\frac{0.0014 - 0.002}{\sigma} < Z < \frac{0.0026 - 0.002}{\sigma}\right) \\ &= P\left(\frac{-0.0006}{\sigma} < Z < \frac{0.0006}{\sigma}\right). \end{aligned}$$

Since $-0.0006/\sigma$ et $0.0006/\sigma$ are equi-distant from 0, then

$$P\left(Z < \frac{-0.0006}{\sigma}\right) = P\left(Z > \frac{0.0006}{\sigma}\right) = \frac{1 - 0.9973}{2} = 0.00135.$$

Thus, we want σ, such that $\Phi(-0.0006/\sigma) = 0.00135$. Hence,

$$\frac{-0.0006}{\sigma} = -2.995.$$

Therefore, $\sigma = -0.0006/-2.995 = 0.0002$ inch.

(b) We want

$$\begin{aligned} 0.9973 &= P\left(0.002 - a < X < 0.002 + a\right) \\ &= P\left(\frac{0.002 - a - 0.002}{0.0004} < Z < \frac{0.002 + a - 0.002}{0.0004}\right) \\ &= P\left(\frac{-a}{0.0004} < Z < \frac{a}{0.0004}\right) \end{aligned}$$

Since $-a/0.0004$ and $a/0.0004$ are equi-distant from zero, then

$$P\left(Z < \frac{-a}{0.0004}\right) = P\left(Z > \frac{a}{0.0004}\right) = \frac{1 - 0.9973}{2} = 0.00135.$$

Thus, we want a, such that $\Phi(-a/0.0004) = 0.00135$. Thus,

$$\frac{-a}{0.0004} = -2.995.$$

Thus, $a = (0.0004)(2.995) = 0.001198$. Donc, the specifications should be from 0.000802 inch to 0.003198 inch.